PROBLEMS FROM THE HISTORY OF MATHEMATICS
PROBLEM SET #4

DUE FRIDAY, 2/23/2018

Exercise 1. Find enough terms in the continued fraction of $\sqrt{6}$ to guess the full expansion. Prove that your expansion is a root of the equation $x^2 = 6$ to justify your guess.

Exercise 2. While performing some calculation you encounter the number

$$x \approx 1.184828323.$$

Context suggests that this number might be an approximation to a simple quadratic irrational. Give a potential closed form for x.

Exercise 3. Suppose that $2^p - 1$ is prime. Prove that p is prime.

Exercise 4. In 1638, Descartes noted that the integer

$$198585576189 = 3^2 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 22021$$

can be used to produce the identity

$$(1 + 3 + 3^2)(1 + 7 + 7^2)(1 + 11 + 11^2)(1 + 13 + 13^2)(1 + 22021)$$
$$= 2 \cdot 3^2 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 22021.$$

Is Descartes' number perfect?[1]

Exercise 5. Prove that any odd perfect number has at least 5 prime factors, counting with multiplicity.

Exercise 5.

a. Use the Lucas-Lehmer test to show that M_{13} is prime.
b. *Optional:* Write a program to show that M_{67} is composite. This number is interesting because it is the first number which was known to be composite (Lucas, 1876) for which no factor was known (until Cole, 1903).

[1] This number is the only odd number known to satisfy a relation of this form.